GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

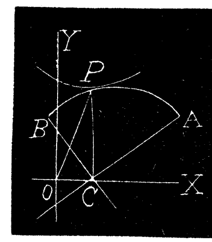
SOLUTIONS OF PROBLEMS.

52. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

If the *center* of a rolling ellipse *move in a horizontal line*, determine the surface on which the ellipse rolls.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.

Let BPA be a quadrant of the ellipse semi-axes AC, and BC, O the position of the center when BC coincides with OY, and $\angle BCP=\theta$. Then



$$PC=y=\frac{ab}{\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}}=\frac{b}{\sqrt{1-e^2\sin^2\theta}}.$$

$\therefore$ The ellipse rolls on the inner surface of the cylinder

$$y^2+z^2=\frac{b^2}{1-e^2\sin^2\theta}.$$

When $e=0$, this becomes $y^2+z^2=b^2$.

To find the abscissa of the point of contact, we have, since arc $PB=$ arc PG,

$$ds=\sqrt{r^2d\theta^2+dr^2}=\sqrt{y^2d\theta^2+dy^2} \text{ since } PC=r=y;$$

$$\text{also } ds=\sqrt{dx^2+dy^2}.$$

$$\therefore \sqrt{dx^2+dy^2}=\sqrt{y^2d\theta^2+dy^2}.$$

$$\therefore dx=yd\theta, \text{ or } x=\int yd\theta=\int\frac{bd\theta}{\sqrt{1-e^2\sin^2\theta}}=b\mathbf{F}(e,\ \theta).$$

When $e=0$, $x=b\theta$.

[No other solution of this problem was received. EDITOR.]

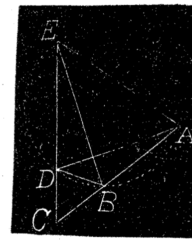
53. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.

A pole, a certain length of whose top is painted white, is standing on the side of a hill. A person at A observes that the white part of the pole subtends an angle equal to α

and on walking to B, a distance a, directly down the hill towards the foot of the pole the white part subtends the same angle. What is the length of the white part, if the point B is at a distance b from the foot of the pole?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.

Let DE be the length painted white; then a circle will pass through A, B, D, E. Let $\angle EAD=\angle EBD=\alpha$, $AB=a$, $BC=b$, $\angle DAB=\angle DEB=\theta$, $\angle ABE=\angle ADE=\varphi$, $DC=y$, and $DE=x$.



Then $(x+y)y=(a+b)b$ .... (1).

$AE:a=\sin\varphi:\sin(\alpha+\theta+\varphi)$, $x:AE=\sin a:\sin\varphi$.

$$\therefore x=\frac{a\sin\alpha}{\sin(\alpha+\theta+\varphi)} \quad \text{........................ (2)},$$

$b:x+y=\sin\theta:\sin(\alpha+\varphi)$ .... (3),

$(x+y):a+b=\sin(\alpha+\theta):\sin(a+\varphi)$ .... (4).

Eliminating θ between (3) and (4),

$$\left\{\frac{(x+y)^4}{(a+b)^2}-\frac{2b(x+y)^2\cos\alpha}{a+b}+b^2\right\}\sin^2(\alpha+\varphi)=(x+y)^2\sin^2\alpha \quad \text{........ (5)}.$$

Eliminating θ between (2) and (3),

$$[\{b^2x^2-x^2(x+y)^2\}^2+4a^2b^2x^2(x+y)^2\sin^2\alpha]\sin^4(\alpha+\varphi)$$
$$-2a^2\sin^2\alpha(x+y)^2\{b^2x^2+x^2(x+y)^2\}$$
$$\sin^2(\alpha+\varphi)+a^4(x+y)^4\sin^4\alpha=0 \quad \text{................ (6)}.$$

Eliminating $\sin(\alpha+\varphi)$ between (5) and (6) we get an equation in x and y which with (1) gives us the value of x.

Solved with result in terms of EC by *A. H. HOLMES*, and *FREDERICK R. HONEY*.

PROBLEMS.

---

58. Proposed by I. J. SCHWATT, Ph. D., Instructor in Mathematics, University of Pennsylvania, Philadelphia, Pennsylvania.

1. The point of intersection K_a' of the tangent drawn to the circumcircle about the triangle ABC at A and the side BC is harmonic conjugate to K_a with respect to BC. (K_a is the point where the symmedian line through A of the triangle ABC meets the side BC.)

2. The point K_a' is the center of the Apollonius circle passing through A of the triangle ABC.

3. Grebes point is on the line joining the middle point of any side of a triangle with the middle point of the altitude to this side.